

Multifractal scaling of electronic transmission resonances in perfect and imperfect Fibonacci δ -function potentials.

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We present here a detailed multifractal scaling study for the electronic transmission resonances with the system size for an infinitely large one dimensional perfect and imperfect quasiperiodic system represented by a sequence of δ -function potentials. The electronic transmission resonances in the energy minibands manifest more and more fragmented nature of the transmittance with the change of system sizes. We claim that when a small perturbation is randomly present at a few number of lattice sites, the nature of electronic states will change and this can be understood by studying the electronic transmittance with the change of system size. We report the different critical states manifested in the size variation of the transmittance corresponding to the resonant energies for both perfect and imperfect cases through multifractal scaling study for few of these resonances.

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I. INTRODUCTION

The discovery of quasiperiodic phase in Al-Mn alloys by Sehechman *et. al* [1] led to a great interest in understanding the generic properties of such systems from electronic structure, transport and phonon dynamics point of view. The simplest possible choice being to study an one dimensional quasiperiodic system. Merlin and co-workers [2,3] first studied the Fibonacci superlattices where the unusual fractal like structure in the spectral properties giving rise to a characteristic features in experiments. Thus the theoretical understanding of such one dimensional quasiperiodic systems and the recent advances of molecular-beam-epitaxy (MBE) make these systems ideal candidates to study and to carry out experiments on transport properties [3] of these systems. Electronic states of one dimensional Fibonacci quasiperiodic systems have been attempted in the recent past [1–8,10–12] to explore some global characteristics in them. It is well known that for such one dimensional quasiperiodic systems the spectrum is a cantor set and has self-similar structure which results from the hierarchy of Fibonacci lattices [4–11]. Most of these previous works have mainly concentrated on studying critical nature of energy bands as well as the nature of wave function from localization point of view for isolated chains. But less attention has been given to the study of the transmittance in detail to look for localization and delocalization aspects through some rigorous scaling analysis [11,12,22,24]. Apart from this, most of the calculations have been performed starting from the tight-binding Anderson Hamiltonian where one models quasiperiodicity either through the diagonal on-site energy or through the off-diagonal hopping

term. However, the actual quasiperiodic potential in the Schrödinger's equation may not be possible to decouple into a diagonal term which will only retain quasiperiodicity and a constant hopping term and vice versa. So one should expect that some of the physical features one finds in the numerical investigation through the continuous models may explain more closely the experimental situation. It is possible to fabricate semiconductor heterostructure with a variety of potential profiles along the growth directions, such as rectangular and parabolic ones in resonant tunneling devices and saw-tooth potential in delta doped layers [13]. Apart from that, it has been realized in the recent past that in some of the thin quasi one dimensional wires, the variation of the potentials at the impurities is much larger than that of the energies involved by the weak electric field; the Kronig-Penny model is then the natural choice for a relevant model to start with.

In recent years, many authors have attempted the problem of studying the scattering of free electrons by an array of δ -function potentials to investigate the electronic states and transport properties of one dimensional disordered systems, quasiperiodic sequences, incommensurate systems [14–19] and in particular the transport properties of quasiperiodic superlattices [13]. Here we have attempted the study of size dependence of transmittance through the multifractal scaling analysis. We have focussed our attention in characterising the transmittance resonances in the different energy minibands that one observes in an array of one dimensional Fibonacci δ -function potentials. This has been done in the same spirit where in a disordered chain one studies the transmittance vs energy plot having signature of

exponentially localized states along with a few stochastic resonances also known as Azbel resonances [14,15].

The strong variation of transmittance with system size gives an indication of the non-trivial nature of transmittance for the various resonant energies. Based on a detailed numerical investigation of the fluctuating pattern of the transmittance within the framework of multifractal scaling of transmittance with system size, it is possible to infer on the delocalization and localization aspects for both perfect and imperfect one dimensional quasiperiodic systems respectively. In this work we will see that both the perfect and imperfect realisations resemble some novel size variation which is intermediate between the so called exponential decay and the homogeneous Blöch like contributions. We claim that many of these are resonant states are of *nearly critical* nature instead of showing either exponential decay or homogeneous spread along the chain. We have made a rigorous study for the electronic transmittance in imperfect Fibonacci system to verify whether a minor change in the potential in a perfect chain can destroy the critical nature of the states altogether leading to a complete localization of electronic states. Recently, some works in the area of imperfect quasiperiodic chains have been reported [23] claiming the instability of the spectrum of a Fibonacci superlattice due to introduction of a single defect. We have seen that for many of the localized states the strong spatial fluctuations in the transmittance will exhibit nearly self-similar features like critical states upto a large length scale followed by a long-tailed oscillatory pattern. However, many of the resonant critical states due to the presence of imperfectness also change but show some fluctuations reflecting some kind of instability in the α - $f(\alpha)$ spectrum subject to large size variation.

II. BRIEF OUTLINE OF THE MODEL

Let us consider the dynamics of electrons in a Fibonacci quasiperiodic array of δ -function potentials. We know that a Fibonacci sequence S_j of order j is obtained by j successive applications of the transformations :

$$S_{j+1} \leftarrow S_j \oplus S_{j-1} \quad (1)$$

$$A \rightarrow AB$$

$$B \rightarrow A$$

Where $S_0 = A$ and $S_1 = AB$ and the symbol \oplus means the operation of composition. For a very large but finite j one can generate an infinitely large array of δ -function potentials in the Fibonacci sequence where the following relations hold:

$$F_{j+1} = F_j + F_{j-1} \quad \text{and} \quad (2)$$

$$\lim_{j \rightarrow \infty} \left(\frac{F_{j-1}}{F_j} \right) = \left(\frac{\sqrt{5}-1}{2} \right) \quad (3)$$

F_j being the number of atoms in the sequence S_j . Recently Sánchez *et. al.* [9] have made an explicit derivation of the generalized Poincare map and calculated the electronic transmittance due to scattering from a lattice with Fibonacci [9] and correlated disorder potentials within the Kronig-Penny model [9]. In this paper we have based our numerical computations of electronic transmittance using the same Poincare map analysis having a very simple and straight forward mathematical structure. This algorithm has been numerically verified to be very stable and suitable for doing calculation for very large system size in one dimensional systems. We illustrate here only a brief outline of the model. A detailed derivation of the generalized Poincare map *etc.* will be found in the ref. [9]. We concentrate on the Fibonacci model for an electron interacting with lattice potentials of the following form.

$$V(x) = \sum_{n=1}^N \Gamma_n \delta(x - x_n) \quad (4)$$

We have chosen the position of the δ -function potentials to be regularly placed (*i.e* $x_n = na, a = 1$) with the strength Γ_A and Γ_B for A and B type of atom respectively. The Schrödinger's equation is then given by (using $\hbar^2 = 2m = 1$),

$$\left[-\nabla^2 + \sum_{n=1}^N \Gamma_n \delta(x - x_n) \right] \Psi(x) = E \Psi(x) \quad (5)$$

One can now introduce reflection and transmission amplitude R_N, T_N respectively through the following relationship:

$$\Psi(x) = e^{ikx} + R_N e^{-ikx} \quad x < 1 \quad (6)$$

$$\Psi(x) = T_N e^{ikx} \quad x \geq N \quad (7)$$

The transmission amplitude T_N after N sites is given by the recurrence relation,

$$A_N = (\alpha_N + \frac{\alpha_{N-1}^* \beta_N}{\beta_{N-1}}) A_{N-1} - (\frac{\beta_N}{\beta_{N-1}}) A_{N-2} \quad (8)$$

where

$$A_N = \frac{1}{T_N^*} \quad (9)$$

$$\alpha_N = \left(1 - i \frac{\Gamma_N}{2k} \right) e^{ik} \quad (10)$$

$$\beta_N = -i \left(\frac{1}{2k} \right) \Gamma_N e^{-ik} \quad (11)$$

The initial conditions $A_0 = 1$ and $A_1 = \alpha_1$ are sufficient to determine the transmission amplitude T_N completely for an arbitrary size *i.e* for a of large number of δ -function potentials. The transmittance t_N is given by

$$t_N = T_N^* T_N = \left(\frac{1}{A_N} \right) \left(\frac{1}{A_N} \right)^* \quad (12)$$

III. MULTIFRACTAL SCALING ANALYSIS FOR THE VARIATION OF TRANSMITTANCE WITH SYSTEM SIZE

In the recent past, Thakur *et. al.* made some electron localization studies in an aperiodic potential where the variation of transmittance with the system size has been investigated [22]. It has been established that this variation of the transmittance with the increase of system size in an infinitely large system exhibits some intrinsic global features which can be fully captured by the multifractal scaling analysis [20]. The same scaling aspects have also been used for the Azbel resonance problem [15] in one dimensional random chain as well as in the study of extended and the critical states in the vicinity of dimer resonance energy in the random dimer problem [24]. Following this line of approach, our motivation in the present work is to examine the electronic transmission of different resonant characters seen in the transmittance versus energy diagram of a quasiperiodic Fibonacci superlattices. We observe that it is possible to explain the dependence of the transmittance with the system size through the multifractal scaling aspects of the transmittance. Here we outline the mathematical content of the multifractal scaling for electronic transmittance only because of its direct connection with our computation and its relation to some of the typical transport features manifested through some global properties in the different energy minibands in our calculations. We will discuss the same in the context of the multifractal scaling algorithm developed by Chhabra and Jensen in brief [21]. This algorithm has been successfully used in analyzing the crossover states in generalized Aubry model, random dimer model and also in many other contexts in the literature [22,24,25]. The multifractal formalism has been developed essentially to describe the statistical properties of some measure in terms of its distribution of the singularity spectrum $f(\alpha)$ corresponding to its singularity strength α . In our calculation we take the normalized transmittance P_i as the required measure which is given by

$$P_i = \frac{T_i}{\sum_{i=1}^N T_i}$$

where T_i is the transmittance from one end of the system upto the i -th segment when the length of the chain is divided into N equal small segments such that the transmittance for a given size is obtained by always increasing the previous size by adding the same number of atoms. According to Chhabra and Jensen if we define the q -th moment of the probability measure P_i by the following expression:

$$\mu_i(q, N) = \frac{P_i^q}{\sum_{i=1}^N P_i^q}$$

then a complete characterization of the fractal singularities can be made in terms of the function $\mu_i(q, N)$. Hence

one can derive the spectrum of fractal singularities defined by, $(\alpha, f(\alpha))$. The expression for $f(\alpha)$ is given by,

$$f(\alpha) = \lim_{N \rightarrow \infty} -\frac{1}{\log N} \sum_{i=1}^N \mu_i(q, N) \log \mu_i(q, N) \quad (13)$$

and the corresponding singularity strength α is obtained as

$$\alpha = \lim_{N \rightarrow \infty} -\frac{1}{\log N} \sum_{i=1}^N \mu_i \log P_i \quad (14)$$

In this regard, the specific nature of electronic transmittance, reflected through α_{max} *i.e* large negative q , α_{min} *i.e* for large positive q , and also through $f(\alpha_{max})$ and $f(\alpha_{min})$ values, can be inferred by the following characteristics for large $N(N \rightarrow \infty)$:

1. Extended nature : $\alpha_{min} \rightarrow 1$, $f(\alpha_{min}) \rightarrow 1$, $\alpha_{max} \rightarrow 1$, $f(\alpha_{max}) \rightarrow 1$.
2. Localized nature : $\alpha_{min} \rightarrow 0$, $f(\alpha_{min}) \rightarrow 0$, $\alpha_{max} \rightarrow \infty$, $f(\alpha_{max}) \rightarrow 1$.
3. Critical nature : α versus $f(\alpha)$ curve will show a tendency to converge onto a single curve as $N \rightarrow \infty$.

Ideally for extended nature the α versus $f(\alpha)$ will contract eventually to $(1, 1)$ whereas for critical nature the α versus $f(\alpha)$ curve, with the increase of system size, may sometimes show minor fluctuations as well.

IV. RESULTS AND DISCUSSION

In this section, we discuss the results for the electronic transmittance of both the perfect and the imperfect Fibonacci superlattices through solving the Eq.(5) making use of the mathematical relations Eqs.(8-12). Here we mainly focus our attention on the electronic transmission resonances in different energy minibands and have investigated whether the resonances survive when we increase the system size drastically by plotting the transmittance versus energy plots for various system sizes. As stated earlier, our aim is to examine whether the delocalized characteristics of the resonances show some interesting universal properties for such quasiperiodic potentials even when subject to a large size variation. We finally claim that our numerical studies of multifractal scaling for the variation of transmittance with system size capture most of the generic features of electronic states and transport in the Fibonacci quasiperiodic superlattice within the Kronig-Penny model.

A. Results for a perfect Fibonacci sequence

In this subsection we discuss the results for a perfect Fibonacci quasiperiodic potential. In Fig.1 we have shown the transmittance T versus E plots for $\Gamma_A = 2$ units and $\Gamma_B = 1$ units for three different system sizes, $N = 75025$ (lower curve), $N = 196418$ (middle curve), and $N = 317811$ (topmost curve). One can clearly see that as the system size increases transmission envelope fragments more and more. It's also true that at any stage, in general, the T versus E plot has a fragmented shape. These characteristic features are due to the critical nature of the energy spectrum of the Fibonacci quasiperiodic system. We now choose arbitrarily a transmission resonance corresponding to a small region of energy mini-bands. This resonance can be identified more clearly if the T versus E plot within this regime can be checked for a very fine energy mesh $\sim 10^{-7}$ or even less such that a resonance within this fine energy resolution will exhibit its signature in the T versus N plot. In Fig.2(a) we have chosen $E = 20.091859999603$ units as one of such resonant energies and have shown the variation of the transmittance T with system size N . The T versus N plot clearly shows highly oscillatory pattern having a very complicated structure and even for variation of size from 10^5 to 7×10^5 no. of atoms or layers it retains the same global shape in its pattern. We have carried out the multifractal scaling of the transmittance with its system size to analyze this spatial pattern in Fig.2(c). Also in Fig.2(b) we have shown another plot of T versus N for 2×10^5 to 3×10^5 no. of atoms or layers for a resonant energy $E = 3.204406000$ units. We see that the oscillatory pattern in Fig.2(b) seems to be different from that of Fig.2(a) altogether. In Fig.2(c) the $\alpha - f(\alpha)$ plot shows a small inward contraction with the increase of system size from $N = 196418$ to $N = 317811$ no. of atoms or layers and then from $N = 317811$ to $N = 514229$ no. of atoms or layers. The deviation of α_{min} and α_{max} values with the size variation is much less and is ~ 0.01 . We identify them by saying *nearly critical resonant states*. In Fig.2(d) the α versus $f(\alpha)$ plot corresponding to the variation of the transmittance T with system size N in Fig.2(b), have been shown. Here we see that as the system size has been increased from $N = 75025$ no. of atoms or layers to $N = 196418$ no. of atoms or layers the curve deviates from the initial position as a whole but when the system size has been increased to $N = 317811$ no. of atoms or layers, the curve comes in between the two previous positions. However, here the amount of deviation of α_{max} values are ~ 0.025 to 0.040 and $\alpha_{min} \sim 0.01$ to 0.02 . One knows that for a localized state with the increase of system size α_{max} values systematically goes out but here for the size $N = 317811$ it comes inside. This is the signature of critical state showing that it's neither localized nor extended but have self-similar pattern for the size variation upto an infinitely large system size. We identify this resonant state as *critical resonant state*.

. We now consider set of parameters for the potentials, namely, $\Gamma_A = 3.0$ units and $\Gamma_B = 1.0$ units and check whether here also one finds some typical resonant states. In Fig.3(a), the T versus N plot has been shown for the resonant energy $E = 6.07038530468009$ units. The variation of the transmittance (T) with system size (N) manifests a highly fragmented and oscillatory behavior from 5×10^5 to 832040 no. of atoms or layers. We also choose another resonant energy $E = 15.0532885305118$ units for the same potential parameters Γ_A and Γ_B where the T versus N plot shows clearly some oscillatory as well as fragmented pattern. However, here, the overall shape approximately gives a periodic oscillation. The $f(\alpha)$ versus α plot corresponding to the variation of transmittance (T) with system size (N) in Fig.3(a) has been shown in Fig.3(c). Note that as we increase the system size from $N = 75025$ to $N = 317811$ both the α_{max} and α_{min} values change by very small amount ~ 0.01 or less. As we increase the size to $N = 514229$, the curve begins to deviate outwards but when we increase the size from $N = 514229$ to $N = 832040$ no. of atoms or layers the curve then again shows an inward contraction. So we see here that the qualitative behavior is much like a resonant state in Fig.2(c) and this can be identified as a critical resonant state as well. Next we analyze the α versus $f(\alpha)$ plot in Fig.3(d) for the resonant state corresponding to the energy $E = 15.0532885305118$ units for which the variation of transmittance with system size has been shown in Fig.3(b). Here we notice that, as we increase the system size from $N = 196418$ to $N = 317811$ no. of atoms or layers the curve undergoes an inward contraction. Next with the subsequent increase of system sizes upto $N = 1346269$ no. of atoms or layers, the curves systematically contracts as we increase the system size more and more, the curves show a relative tendency to contract by very lesser amount. We identify this to be qualitatively similar to the resonant states as depicted in Fig.2(c). So we again identify it to be a *nearly critical resonant state* and it is in some sense different from the critical resonant shown in Fig.2(b) and Fig.3(c).

We think these two typical resonant states, namely, critical and nearly critical can be identified for different choice of potentials also. It's also true that they are of two distinct nature which one can differentiate through the α versus $f(\alpha)$ plots. These resonant states exist for different energy regimes and also for any choice of Γ_A and Γ_B . This particular aspect is true since we have chosen the resonant energies in Fig.2(a) and Fig.2(b) or in Fig.3(a) and Fig.3(b) in completely different energy regimes arbitrarily.

B. Results for an imperfect Fibonacci sequence

Now we go over to analyze the effects of disorder present at random in any layer. The amount of perturbation in the potential parameters Γ_A and Γ_B have been

considered 5% at random in any layers through perturbing 2% of the atoms or layers in the system as a whole.

In Fig.4(a) we have shown the transmittance(T) versus system size(N) plot for the resonant energy $E = 6.07062311828030$ units corresponding to starting Fibonacci potentials $\Gamma_A = 3.0$ units and $\Gamma_B = 1.0$ unit and then a fractional change in the potentials $f = 0.05$ have been considered randomly at any layer where 2% of the layers have been perturbed as a whole. One can see that the transmittance amplitudes look like different patches of spikes of many different heights. The variation of transmittance with system size(N) has been shown from 10^6 to 1346269 no. of layers. Note that it looks neither like extended nature or exponentially localized but apparently the profile of the spikes seem to have some power law decay. We have checked through the multifractal scaling analysis whether the resonant states would show some systematic decay or not.

In Fig.4(b), the plot of $f(\alpha)$ versus α corresponding to the variation of transmittance with system size in Fig.4(a), have been shown. We have shown α versus $f(\alpha)$ curves corresponding to the size variation starting with the initial size 832040 no. of atoms or layers and then successively increasing no. of atoms or layers by 10^5 no. of atoms or layers. One can clearly see in Fig.4(b) that α versus $f(\alpha)$ curves show some instability even for the variation of transmittance beyond 10^6 no. of atoms or layers. This instability in the α versus $f(\alpha)$ plot is due to the effect of disorder in the Fibonacci quasiperiodic layers. However, if one looks at the overall variation of the transmittance with system size, from 10^6 to 1346269 no. atoms or layers then it shows the decay but of very complicated form and this is also captured in the outermost α versus $f(\alpha)$ plot corresponding to the variation of transmittance T for the system of size from 832040 to 1346269 no. of atoms or layers eventually. One can see that this is the typical long-tailed fluctuations which we have observed due to the interplay of the effect of disorder and the resonant nature of critical state in the ideal system. Since this is a state which shows a size variation in between critical and exponentially localized state, one can identify this resonant state, due to the presence of imperfectness in the system, to be more closely like *critical resonant state*. Next we have shown, in Fig.5(a), the variation of transmittance with system size for a resonant state corresponding to the energy $E = 4.3987600006124$ units and for the values $\Gamma_A = 2.0$ units, $\Gamma_B = 1.0$ units. It's a typical localized state with fluctuations throughout but having less fluctuations in the tail part as compared to that in the beginning, *i.e.*, around 50,000 to 2×10^5 no. of atoms or layers. In Fig.5(b), the α versus $f(\alpha)$ plots corresponding to the variation of transmittance with system size as shown in Fig.5(a), for $N = 317811, 514229$ and, $N = 832040$ no. of atoms or layers, have been shown. It's quite clear that with the increase of system size the curves begin to move outwards reflecting the very nature of a localized state having an overall systematic decay. In Fig.5(c) we have shown the plot of transmittance

with the system size N for an imperfect superlattice with $E = 20.88552401746$ units. This is also a resonant state with strong fluctuations throughout which arises because of the presence of disorder in the system. In Fig.5(d) we have shown the α versus $f(\alpha)$ plot corresponding to the variation of the transmittance(T) with system size (N) in Fig.5(c). Note that, as the system size has been increased from $N = 196418$ to 317811 no. of atoms or layers, the curves undergo an outward shift but as we increase the system size from 317811 to 514229 no. of atoms or layers it again goes in between the two previous positions. This sort of instability reflected in the α versus $f(\alpha)$ is due to the presence of imperfectness coupled with the long-ranged quasiperiodic order in the system. Since these states do not show any systematic decay either in the $\alpha - f(\alpha)$ plot, and show some kind of critical nature, we identify them more closely like *critical resonant states* also. Therefore, to paraphrase, the manifestation of the effect of disorder on the resonant critical states, apart from the existence of having systematic decay, has been found to be twofold: firstly, it has long-tailed fluctuations and secondly having sustained strong fluctuations even when subject to large size variations as mentioned above.

V. CONCLUSION

In this work we explore the nature of electronic states and transport in one dimensional δ -functional Fibonacci potentials from a rigorous study of electronic transmission resonances corresponding to the different energy minibands. In this regard, although some studies of transmittance have been reported in the past, our numerical study of the size dependence of the same is directly associated with the characterization of the nature of electronic states and transmittance using the multifractal scaling aspects which to our knowledge has not yet been investigated adequately till date. We should point out the fact that many of the resonances show strong oscillations with the variation of the system size and the oscillating pattern continues to survive till the width of the minibands, in which it belongs, goes to zero and an energy gap opens up there. In a perfect Fibonacci case, critical behavior of the transmittance with system size has its origin from the typical nature of the Fibonacci potential and at least a few of them can be distinguished from the others. More specifically, here, the clear distinction between the different critical resonant states have been possible with the help of multifractal scaling analysis. On the other hand, we have considered the imperfect situation to be a case where the imperfectness appears randomly as a homogeneous substitutional disorder in the different lattice points causing only minor changes in the Fibonacci potentials which may be present *e.g.* in an experimental realisation of Fibonacci superlattice.

In the imperfect case we have examined that this

may induce localization effects which may be manifested through some non-exponential spatial decay of transmittance and the resonant states in the perfect situation will appear again more closely like critical resonant states but with some modified nature.

The long-tailed oscillatory pattern of the transmittance shows a very slow spatial decay and it retains its nearly self-similar pattern even if one drastically changes the system sizes. Although the overall pattern of the oscillations eventually show decaying fluctuations, the decay is not uniform— indicating an interplay between the long-ranged quasiperiodic order and the disorder an electron experiences with the increase of system size. The other manifestation of the interplay between long-ranged quasiperiodic order and substitutional disorder is the instability of the resonant state in the variation of transmittance with the increase of system size. This has been also reflected in the α - $f(\alpha)$ spectrum in our numerical studies. We, therefore, hope that it will be interesting to search for both non-exponential localization and critical properties of the electronic transmittance through designing some controlled experiments in this direction. We believe that our work will shed some more light in understanding the theoretical notion of the novel scaling properties for the electron localization and delocalization aspects directly in the size dependence of the conductance in Fibonacci quasiperiodic systems.

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FIG. 1. Tranamittance T versus energy E plots for $\Gamma_A = 2$ units and $\Gamma_B = 1$ units . $N = 75025$ no. of atoms or layers (lower curve , $N = 196418$ no. of atoms or layers (middle curve), $N = 317811$ no. of atoms or layers (topmost curve).

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FIG. 2. (a) Transmittance T versus system size N for a pure Fibonacci lattice with $\Gamma_A = 2$ units, $\Gamma_B = 1$ unit and $E = 20.0918599999603$ units.

(b) Transmittance T versus system size N for a pure Fibonacci lattice with $\Gamma_A = 2$ units, $\Gamma_B = 1$ unit and $E = 3.20440600$ units.

(c) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 2(a) but for different system sizes having maximum no. of atoms or layers $N = 196418$, (line), $N = 317811$ (dashed line) and for $N = 514229$ (smaller dashed line), where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

(d) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 2(b) but for different system sizes having maximum no. of atoms or layers $N = 75025$, (line), $N = 196418$ (dashed line) and for $N = 317811$ (smaller dashed line), where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

FIG. 3. (a) Transmittance T versus system size N for a pure Fibonacci lattice with $\Gamma_A = 3$ units, $\Gamma_B = 1$ unit and $E = 6.07038530468009$ units.

(b) Transmittance T versus system size N for a pure Fibonacci lattice for the same set of potential parameters as in Fig. 3(a) but for energy value $E = 15.0532885305118$ units.

(c) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 3(a) for different system sizes having maximum no. of atoms or layers $N = 75025$ (line), $N = 317811$ (dashed line), $N = 514229$ (smaller dashed line) and $N = 832040$ (dotted line), where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

(d) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 3(b) for different system sizes having maximum no. of atoms or layers $N = 196418$ (line), $N = 317811$ (dashed line), $N = 514229$ (smaller dashed line), $N = 832040$ (dotted line) and for $N = 1346269$ (dash-dotted line), where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

FIG. 4. (a) Transmittance T versus system size N for an imperfect Fibonacci lattice for $\Gamma_A = 3.0$ units, $\Gamma_B = 1$ unit for $E = 6.07062311828030$ units. The fractional change $f = 0.05$ has been considered randomly at any layer or atom (say i) so that the new $\Gamma_i = \Gamma_i + f * \Gamma_i$. The concentration of such defects present in the lattice is $c = 0.02$

(b) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 4(a) for different system sizes. The data set have been considered starting with size having no. of atoms or layers $N = 832040$ and adding 10^5 no. of atoms or layers (line), adding 2×10^5 no. of atoms or layers (dashed line), 3×10^5 no. of atoms or layers (smaller dashed line), 4×10^5 no. of atoms or layers (dotted line) and 5×10^5 no. of layers (dash-dotted line) with steps of 5 layers in each case.

FIG. 5. (a) Transmittance T versus system size N for an imperfect Fibonacci lattice for the set of parameters $\Gamma_A = 2.0$ units and $\Gamma_B = 1.0$ unit for $E = 4.39876000006124$ units. The fractional change $f = 0.05$ has been considered randomly at any layer or atom so that the new $\Gamma_i = \Gamma_i + f * \Gamma_i$. The concentration of such defects present in the lattice is $c = 0.02$.

(b) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 5(a) for different system sizes. The data set have been considered starting with size having maximum no. of atoms or layers $N = 317811$ (line), $N = 514229$ (dashed line) and $N = 832040$ (smaller dashed line) no. of atoms or layers, where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

(c) Transmittance T versus system size N for an imperfect Fibonacci lattice for the same set of potential parameters as in Fig. 5(a) but for $E = 20.88552401746$ units.

(d) $f(\alpha)$ versus α plot for the same set of parameters as in Fig. 5(c) for different system sizes. The data set have been considered starting with size having maximum no. of atoms or layers $N = 196418$ (line), $N = 317811$ (dashed line) and $N = 514229$ (smaller dashed line), where in each case starting size is 5 no. of atoms or layers and the rest is obtained increasing 5 no. of atoms or layers each time.

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